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Practical Implications of Parabolas

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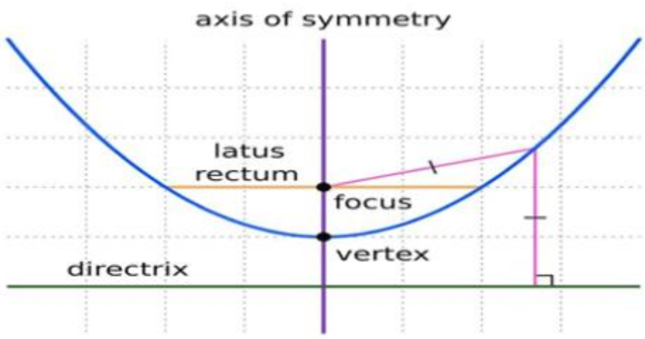
Abstract: This project highlights the significance of parabolas in technology, science, and architecture while examining their various everyday uses. Because they can concentrate sound waves at a particular location, parabolic shapes are essential in the construction of satellite dishes, automobile headlights, and flashlights. This project illustrates mathematics and practical problem-solving structural stability and visual appeal. This project illustrates how mathematics and practical problem solving interact by jamming the uses of parabolas, showing how a straightforward geometric shape can transform technology and enhance everyday life.

Keywords: Axis; Directrix; Focus; Focal length; Parabola; Vertex

INTRODUCTION

A parabola is a planar curve that is mirror symmetrical and roughly U-shaped. It matches numerous superficially diverse mathematical formulations that may all be shown to define the same curves.



In other words, a parabola is the set of all plane points equidistant between a given line L (the conic section directrix) and a given point F not on the line (the focus). Therefore, p = 2a, where a is the distance from the vertex to the directrix or focus, yielding the focal parameter, or the distance between the directrix and the focus. A paraboloid is the surface of revolution that is produced when a parabola is rotated around its axis of symmetry. The parabola's focus on the graph is located at

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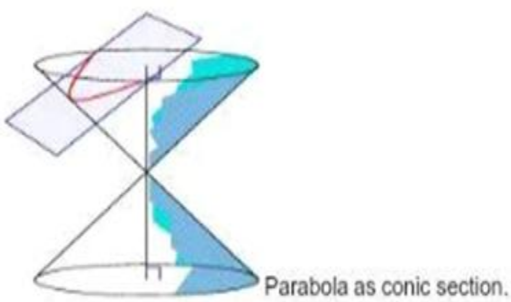
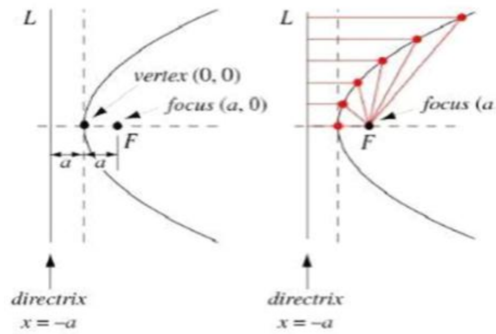
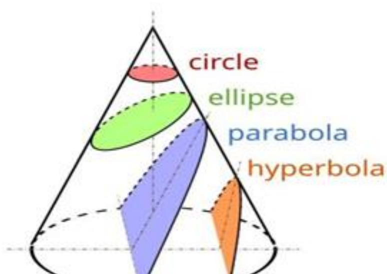
(a,0).”x = -a” is the directrix. The distance between the origin and the directrix, as well as the distance between the origin and the focus, is |a|. Since distance is positive, we take the absolute value. On the curve, any point is represented by the symbol (x, y). The distance d between any two points (x, y) and the focus (a, 0) is equal to the distance between the two points (x, y) and the directrix.





A line (the directrix) and a point (the focus) can be used to characterize a parabola. The primary focus is not the directrix. The parabola is the arrangement of points in that plane that are equally far from the focus and the directrix. When a right circular conical surface meets with a plane parallel to a surface that is tangential to the conical surface, a parabola—also known as a conic section—is created.

With its axis parallel to the y-axis, the graph of a quadratic function $y = ax^2 + bx + c$ (with $a \neq 0$) is a parabola. On the other hand, each of these parabolas represents the graph of a quadratic function. The “axis of symmetry” is the line that splits the parabola through the middle, or the line that is perpendicular to the directrix and passes through the focus. Known as the “vertex,” this is the place where the parabola is most steeply bent and touches its axis of symmetry. The “focal length” is the distance, measured along the axis of symmetry, between the vertex and the focus. The code of the parabola that runs parallel to the directrix and across the focus is known as the “latus rectum.” A parabola can open in any direction, including up, down, left, and right. Since all parabolas are geometrically similar, any parabola may be precisely resized and moved to fit on any other parabola

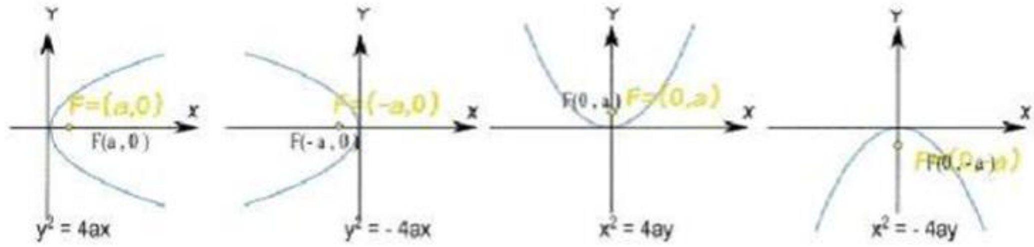


The parabola is a member of the family of conic sections.

Light flowing parallel to the parabola’s axis of symmetry and striking its concave side will be reflected to its focus, regardless of where the reflection occurs on the parabola, assuming the parabola is made of light-reflecting material. However, when light from a point source at the focus is reflected into a parallel “collimated” beam, the parabola stays parallel to the axis of symmetry. The effects of sound and other waves are identical. This reflecting property is the foundation for many practical uses of parabolas. Ballistic missile design, car headlight reflectors, and parabolic antennas and microphones are just a few of the several significant uses for the parabola. In several fields, including physics and engineering, it is widely utilized.

METHOD

Equations of Parabola:



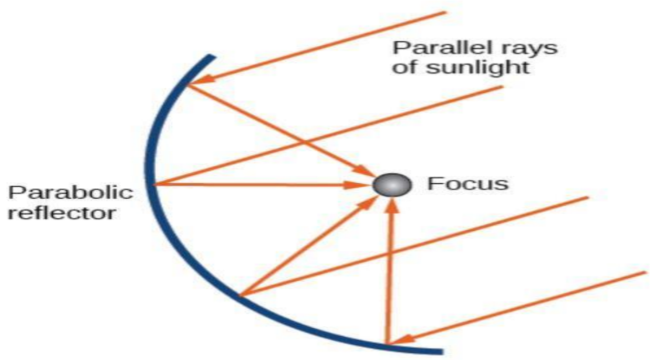
The general conic equation can be used to determine the gender equation of a parabola, which is:
$Ax^2+Bxy+ Cy^2 + Dx + Ey + f = 0$
additionally, the reality that $B^2= 4AC$ for a parabola.
The equation for a generic parabola with a focus point F(u, v) and a directrix represented by ax + by + c = 0 is.

$$\frac{(ax+by+c)^2}{a^2+b^2} = (x-u)^2 + (y-v)^2 \quad (1)$$

RESULT AND DISCUSSION

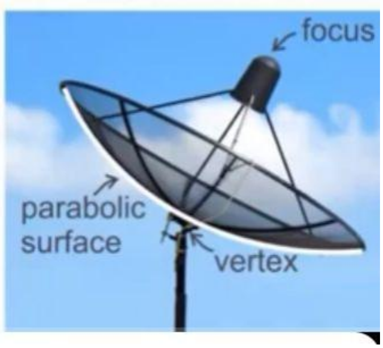
Some applied numericals:
Problem 1:

The focus of a parabolic reflector is positioned 5 cm from its vertex. Determine the equation of the parabola if it is in the conventional form $y^2=4px$, where p is the focal length. Determine the location on the parabola where a light beam parallel to the axis reflects through the focus as well.
Solution.



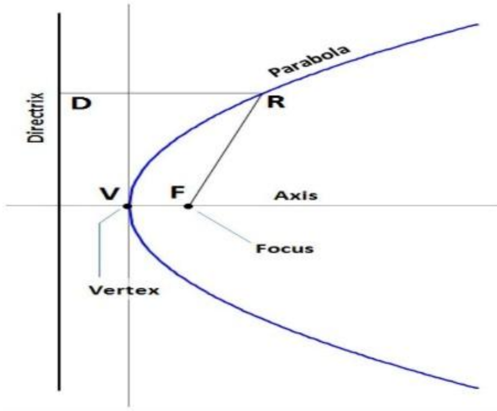
The parabola's equation, given p = 5 cm, is:
$y^2=4px=4(5)x=20x$
The light reflects through the focus at (5,0) for a beam of light parallel to the axis of symmetry. Using x=5, one may find the point on the parabola where the beam reflects:
$y^2=20(5)=100 \Longrightarrow y=\pm 10$
The parabola's equation is $y^2=20x$. The points (5,10) and (5,−10) are where the light reflects.

Problem 2:
A satellite dish is shaped like a parabola, measuring four meters in diameter and one meter in depth. Determine the parabola's equation and focus position.



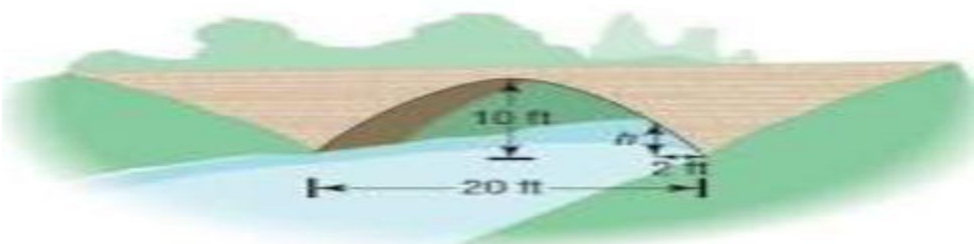
Solution:
The vertex should be placed at the origin. The parabola has the shape $x^2=4py$ and opens upward. On the dish's edge (x=2, y=1):
$2^2=4p(1) \Longrightarrow 4=4p \Longrightarrow p=1$.
$x=4y$ is the equation. The focus is situated one meter above the vertex at p=1.
The parabola's equation is $x^2=4y$, with the focus at (0,1).

Problem 3:
The point of focus of a parabolic reflector is 5 cm above the vertex. Determine the reflector's equation if its opening is 20 cm wide.
Solution:



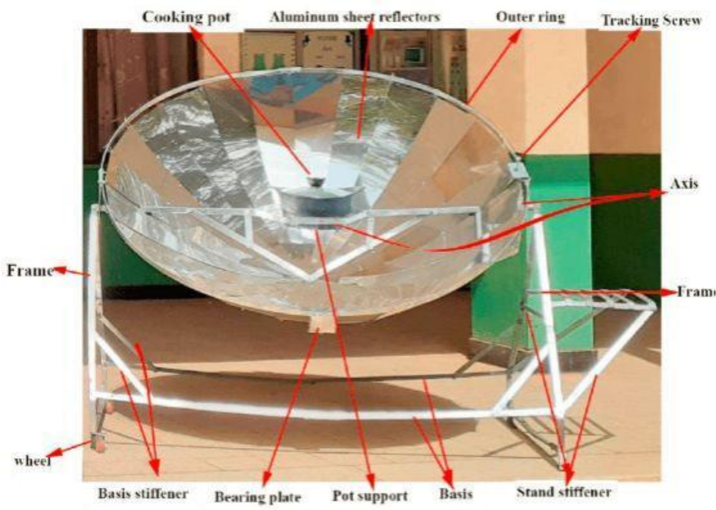
The vertex is at (0,0), while the focus is at (0,5). The equation is $x^2=4py$, where p=5, since the parabola widens upward.
To determine the opening's width, use $y^2=20$, which is the distance between the parabola's vertex and edge.
$x^2=20(5) \Longrightarrow x^2=100 \Longrightarrow x=\pm 10$
$x^2=20y$, width: $2\times 10=20$ cm is the equation.

Problem 4:
The main height of a parabolic arch footbridge is 20 meters, and its span is 100 meters. At a distance of 25 meters from the center, what height is the arch?
Solution:



The equation of the parabola is $y = a(x-50)^2 + 20$.
We may determine the value of 'a' by using the vertex point (50, 20):
$0 = a(0-50)^2 + 20$
$\Rightarrow$ $a = -1/125$
In order to determine the height at x = 25 meters, we substitute:
$y = (-1/125)(25-50)^2 + 20$
$\Rightarrow$ $y = 15$ meters

Problem 5:
The diameter of a parabolic solar cooker is 2 meters. What is the cooker's depth if the parabola's focus is 25 cm from the vertex?
Solution:



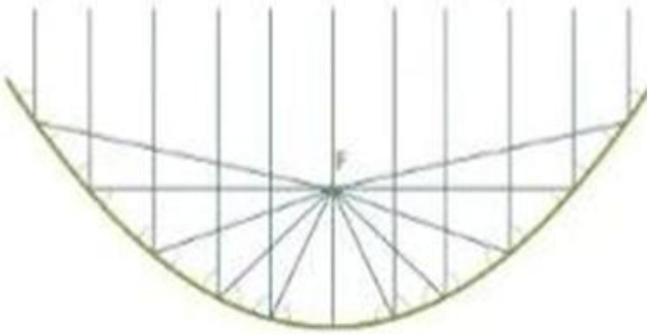
$y = ax^2$ is the parabola's equation. The point of interest is $(0, 1/(4a)) = (0, 0.25)$. Therefore, $1/(4a) = 0.25$, meaning that $a = 1$.
The radius is one meter since the diameter is two meters. $y = 1^2 = 1$ for $x = 1$.
As a result, the cooker is one meter deep.

History of parabola:

Parabolic compass designed by Leonardo da Vinci

Menaechmus wrote the first documented work on conic sections in the fourth century BC. He found a method to use parabolas to solve the cube doubling problem. However, the approach does not satisfy the straightedge and compass assembly requirements. In his book The Quadrature of the Parabola, written in the third century BC, Archimedes used the method of exhaustion to calculate the area bounded by a parabola and the line segment, also known as the "parabola segment." Apollonius, who found numerous characteristics of conic sections, is credited with giving the term "parabola" its name.

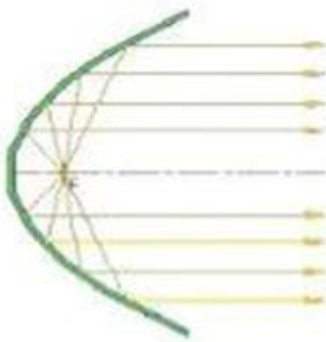
It signifies “application,” alluding to the idea of “application of areas,” which is related to this curve, as Apollonius demonstrated. Papapus made reference to the parabola’s and other conic sections’ focus-directrix characteristic.



Galileo demonstrated how a projectile’s component accelerates uniformly due to gravity in a sequence known as a parabola. Before the reflecting telescope was created, it was already widely recognized that a parabolic reflector could create an image. Many mathematicians, including James Gregory, Marin Mersenne, and René Descartes, presented designs in the early to mid-17th century.
Because parabolic mirrors are difficult to fabricate, Isaac Newton used spherical mirrors instead of parabolic ones when he built the first reflecting telescope in 1668. The majority of contemporary reflecting telescopes, satellite dishes, and radar receivers employ parabolic mirrors (see [1-5]).
Although many ancient structures used circular arches, the parabola’s shape was recognized for its ability to distribute forces efficiently. Over time, parabolic arches became integral to bridge design and modern construction. Ancient siege engines were engineered to use parabolic trajectories to optimize the distance and accuracy of projectiles.
Johannes Kepler, in the early 17th century, noted that objects under gravitational influence often follow parabolic trajectories, contributing to Newton’s laws of motion and universal gravitation. During the Renaissance and later centuries, parabolic designs became crucial in the development of solar concentrators, cars, headlights, and satellite dishes (see [6-9]).

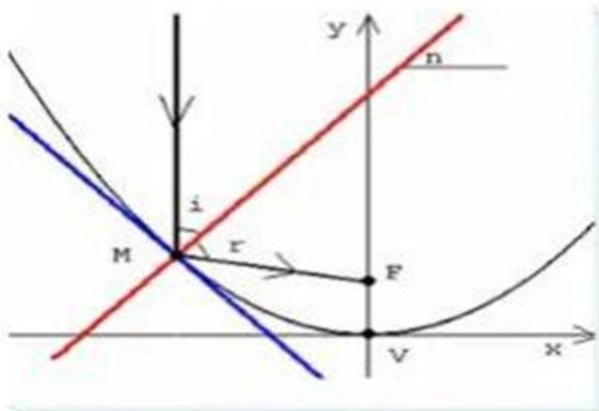
<u>The uses and applications of parabolas:</u>

- The parabolic reflector, a mirror or similar reflective device, focuses light or other electromagnetic waves to a single focal point. Light from a focusable point source, on the other hand, is collimated into a parallel beam.



The principle of the parabolic reflector may have been established in the third century BC by the geometer Archimedes, according to a plot of dubious authenticity. By directing the sun’s rays to burn the Roman ships’ decks, he built parabolic mirrors to defend Syracuse from the Roman fleet. The theory was used for telescopes in the 17th century.

- Car headlights, flashlights, and other devices use the parabolic reflector idea. A parabolic mirror is used to focus the light, and as the light passes through and meets the mirror, it is reflected in straight lines that are parallel to the axis, as seen in the diagram. This explains why the light beam from torches and automobile headlights is so powerful.



- The point where the ray hits the parabolic dish itt's named Point M. At point M, I denotes the angle generated by the incident ray and the normal perpendicular to the parabola's tangent. The angle created by the normal and reflected beams is represented as r. Angles I and r are equal under the law of reflection. All reflected rays from incident rays at various points intercept the y axis of the parabola at the same time. A parabolic microphone's highly directed performance is obtained by focusing sound onto it via a parabolic reflector, which reflects sound but not electromagnetic radiation.

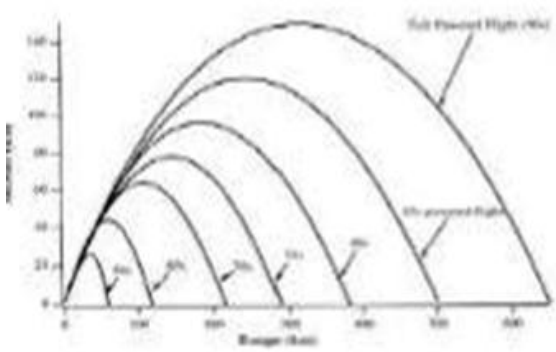
- Parabolic skis:

Typical skis start off with a circular side cut that deforms under load to make a perfect arc. When under load, the parabolic design will bend into a perfect arc in a smooth turn. As a result, the skier needs to exert the least amount of effort to turn flawlessly—just tipping it on edge.

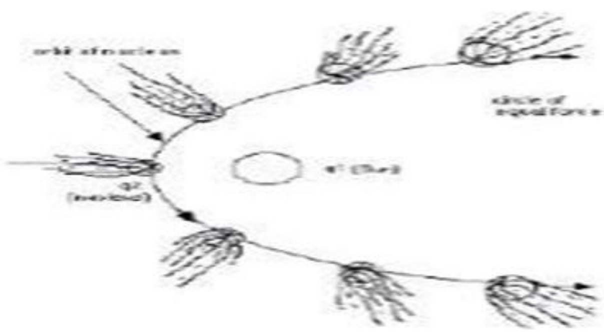
- Water jets, such as those produced by fountains, take the shape of a parabola.

A parabola is formed when a ball is struck in the air and moves apart.Additionally, paraboloids can be seen on the surface of a liquid that is contained within our container and rotated about its center axis. In this instance, the liquid climbs the container walls due to centrifugal force, creating a parabolic surface. The liquid mirror telescope works on this concept. A rectangular container filled with fluid is displayed here; the container is set inside a revolving table, and the fluid inside takes the shape of a parabola as the table revolves.

- In the real world, the trajectory is always a rough idea of a parabola. For example, although the shape is very similar to a parabola at low speeds, air resistance naturally distorts it. However, in ballistics, the form slip is highly twisted and does not resemble a parabola at high speeds.



- Long-period comets' paths are not parabolic because they approach the sun's escape velocity as they pass through the inner solar system. Traveling through the inner solar system, Comet Kohutek showed off its nearly parabolic shape.



CONCLUSION

The parabola, a fundamental shape in mathematics, is widely used in different sectors of real life due to its distinctive geometric and reflective features. Its significance is especially evident in engineering, physics, architecture, and astronomy. Parabolic mirrors and antennas use the focus-directrix feature to concentrate light or signals, enabling technology such as satellite dishes, telescopes, and automobile headlights. Bridges and arches frequently use parabolic shapes to efficiently distribute weight and stress while maintaining structural stability and beauty. In sports and physics, the parabolic trajectory characterizes the motion of projectiles ranging from thrown balls to missiles, which aids in design optimization and motion path prediction. Parabolas are particularly important in creating roller coasters and water fountains because of their aesthetic appeal.

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Conflicts of Interest
The authors declare no conflict of interest.

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