



Paddy Price Prediction using Fuzzy Time Series Model Lee Method for Determination of Crop Insurance Premiums

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Abstract: The price of the paddy has significant fluctuations as BPS mentioned that the average price of dry paddy harvested at the farmer level decreased in February 2022 by 3.2% from January 2022. Hence, crop industry businesses continue to face significant uncertainty risk. The purpose of this study is to discuss the use of the Fuzzy Time Series Model Lee for predicting future paddy prices in order to calculate crop insurance premium using Black Scholes model with cash or nothing put option approach. This is because crop industry is one of the agricultural products that Indonesia is capable of producing in large quantities. As a result, crop insurance should be purchased by farmers to protect against crop yield losses. Aside from that, the price of paddy fluctuates significantly. Therefore to reduce the loss of revenue from reductions in decreasing of crop yield or even crop failure, it needs to provide the insurance based on the paddy prices to protect the paddy prices itself from the large fluctuations at the farmer level. Based on the analysis of this study, generates result for January 2022, February 2022, and March 2022 are 4547.41, 4547.41, and 4701.62 respectively. With the accuracy level is 0.05%. Therefore, the insurance premiums based on the prediction result is 2,775,579. The implication or benefit of this thesis is for the other parties such as farmer.

Keywords: Fuzzy; Lee model; Crop; Insurance.

INTRODUCTION

Indonesia is called an agricultural country because it is able to produce agricultural products in large quantities. This is because most of the population has jobs as farmers. Therefore, Indonesia has great potential in the agricultural sector, one of which is crop industry. However, businesses in the crop industry are still faced with considerable uncertainty risk. These is caused by the pest disease attacks and unpredictable weather such as rainfall and drought that probably generates floods and dryness are the factors of decreasing the crop yield or even crop failure and result in a lot of losses.

According to the data based on Kementan, The fields of crop failure caused by pest disease attacks for period January to October in 2021 and 2022 are 3959 ha and 6218 ha, respectively (Anonymous, 2022). Meanwhile, the impact of floods towards the fields of crop failure with the same

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period in 2021 and 2022 are 81,235 ha and 26,167 ha, respectively. On the other hand, the dryness causing the crop failure of fields within the same periods and year as the pest disease attacks and floods are 6930 ha and 32 ha, respectively. The risk of uncertainty is a burden to farmers. Hence, to protect the losses of crop yield, the farmers should be secured with crop insurance. In Indonesia there is crop insurance product named Asuransi Usaha Tani Padi (AUTP).

It is cover the farmer against land damage. Nevertheless, it is considered by the farmers that it has too expensive of the insurance premiums and the process of claim is complicated (Hidayat, 2019). Besides that, there is also another crop insurance product from outside Indonesia which is developed countries they are China, Iran, and Australia had already used the crop insurances based on the weather index that usually to protect from drought (Hidayat, 2019). To calculate the crop insurance premium requires the method, one of them is Black Scholes method with the put option type. The method of calculating the price of crop insurance premiums in Indonesia based on the rainfall index has also been introduced by Putri (2017) using Black Scholes method, Qosim (2018) using Exponential Distribution method, and Hidayat (2019) using Vine Copula method. Meanwhile, there is also the study of calculate the crop insurance premiums based paddy prices, where it can be known from Primajati (2020) study, where it determine the crop insurance premiums based on the lower selling prices using Barrier options model. It is therefore noteworthy that there are 3 plants seasons of paddy in Indonesia which are main (from November to March), gadu (from April to July), and drought (from August to October) (Vio, 2021). Main season, total of paddy harvest is greater than gadu and drought season. It is because the main season is often occurs in the rainy season, meanwhile gadu and drought season can depend only on rainwater and good irrigation Besides that, the price of the paddy has significant fluctuations as BPS mentioned that the average price of dry paddy harvested at the farmer level decreased in February 2022 by 3.2% from January 2022 (Darmawan, 2022). Although it is not yet the great harvest seasons which are usually occurs in March – April, the prices have fallen. To overcome the uncertainty regarding fluctuations in paddy prices requires forecasting of paddy prices at the farmer level to predict the price for the next three periods. Therefore, to reduce the loss of revenue from reductions in decreasing of crop yield or even crop failure, it needs to provide the insurance based on the paddy prices to protect the paddy prices itself from the large fluctuations in the farmer level. It needs the indemnity limit as a term to calculate the insurance premiums. Then, to determine the indemnity limit, it will use Fuzzy Time Series method.

According to Wang (2015), the Fuzzy Time Series is a development of methods to overcome the weaknesses that existed in the previous forecasting method. The previous forecasting method is called ARIMA. The weakness of the method is that it requires a lot of historical data and requires certain assumptions that must be met. Meanwhile, Fuzzy Time Series (FTS) does not require a large amount of historical data and assumptions in forecasting. FTS itself is a data forecasting that uses the Fuzzy set as the basis for forecasting modeling. This forecasting uses past data patterns and is then processed to forecast future data. In the Fuzzy Time Series method, there are several models, namely the Song, Chissom, Chen, Cheng and Lee models. FTS Lee is a model of the FTS method that developed after Song, Chissom, and Chen. Some researchers have conducted studies using this method. Such as the research conducted by Lestari (2017) with the title The Use of the Fuzzy Time Series Method to Predict Rice Production Results in Majalengka Regency. The methods used are Chen's fuzzy time series and Cheng's fuzzy time series. The results of the study can be concluded that the best accuracy method is to use the Cheng fuzzy time series method with forecasting accuracy reaching 95.76% and forecasting results for 2017 obtained rice production of 677943.6 tons. Patria (2021) with the research title Fuzzy Time Series Application in Predicting the Number of Confirmation Cases of Covid-19 Patients in Indonesia. The researcher used the Chen and Cheng model as a method used to analyze the accuracy of the predicted results of Covid-19 patients in Indonesia. the accuracy is then calculated using the Mean Absolute Percentage Error (MAPE). The accuracy results of each Chen and Cheng model are 12.75% and 14.27% which is great accurate also can be used to predict Covid-19 cases in Indonesia.

Therefore, the author tries to predict the price of paddy using the Fuzzy Time Series Model Lee method and the results of the prediction are used for the indemnity limit then will be used to determine crop insurance premiums using Black Scholes method. Where in fuzzy time series process lengths intervals already regulated in an initial process caused by lengths intervals is crucial in the prediction result, so that the formation of the fuzzy relationship will be appropriate (Handayani, 2015). Methods to determine the prestigious intervals lengths is the average based fuzzy time series.

Option

An option is a contract that grants the holder the right (but not the obligation) to buy or sell a specific asset at a specific price within a specific time frame (Putri, 2017). The price is known as the exercise price or strike price and the time is known as maturity date or expiration date. Options there are Americans and Europeans. Americans option can be exercised at any time until maturity time, meanwhile Europeans option can be exercised at maturity time only.

Black Scholes

Method had been advanced in 1973 by Fisher Black and Myron Scholes, and it is used to determine the value of an option in a stock price contract. The Black-Scholes method makes several assumptions, including that the option used as a reference is the European option, that the volatility (variance price) is constant (fixed) over the life of the option, that the stock used are not dividends, and that taxes and transaction fees are ignored (Kurniawan, 2012). The formula for put option black scholes model European can be written as:

$$P(S, T) = N(-d_2)Ke^{-rT} - N(-d_1)S_T \quad (1)$$

with:

$$d_1 = \frac{\ln\left(\frac{S_T}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T} \quad (2)$$

where:

- S_T : Prediction paddy price at time T
- K : Indemnity limit
- r : Risk free rate
- σ : Volatility of the paddy price
- T : Maturity date (in year)
- $N(d_1)$: Cumulative normal distribution function of d_1
- $N(d_2)$: Cumulative normal distribution function of d_2

Cash or Nothing Option

In black scholes model, cash or nothing where the payoff depends on the outcome of yes or no and typically relates to whether the price of a particular asset will rise above or fall below a certain amount. It means that will get the cash P (money) or nothing at all. Put payoff function is $S_T < K$ and 0 otherwise. Cash or nothing put option formula can be seen as below:

$$CashPut = Pe^{-rT}N(-d_2) \quad (3)$$

where:

- P : Indemnity value (in cash)
- S_T : Prediction paddy price at time T
- K : Indemnity limit
- T : Maturity date (in year)
- r : Risk free rate
- $N(-d_2)$: Cumulative normal distribution function of $-d_2$

Calculation of Return, Volatility, and Standard Deviation

Return expectation is the return expected to occur in the future and is still uncertain, whereas realized return is one of the company's performance measurements and can be used as a basis

determinant of future return expectations and risks (Joharni, 2017). The formula that will be used to calculate the stock price return when $i=1,2,3,\dots,n$ is (Hull, 2009):

$$R_i = \ln \frac{S_i}{S_{i-1}} \quad (4)$$

where:

- R_i : Return at time i
 S_i : Stock price at time i
 S_{i-1} : Stock price before time i

The volatility of a paddy price can be defined as the standard deviation of the paddy's annual return. The formula for volatility is:

$$\sigma = \frac{s}{\sqrt{\frac{1}{k}}} \quad (5)$$

where k is the total months of trading in a year. Meanwhile s is the standard deviation of stock price return can be formed as:

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (R_i - \bar{R}_t)^2} \quad (6)$$

where n is the total of the data, \bar{R}_t is the mean of the data.

Fuzzy Set

Fuzzy set is a class of object with the entire continuum from the membership value. In a fuzzy set, the membership value is 0 to 1, which means that the fuzzy set can represent the interpretation of each value based on its beliefs or decisions and probabilities. A value of 0 means false and a value of 1 means true, but there is a value that is between true and false. In other words, the true value of an object is not only true or false. The membership value is defined based on the rules (Boaisha, 2010) where $1 = i = n$, which n is the total class of fuzzy interval. The value of matrix is 1 if $j = i$, matrix value is 0.5 if $j = i - 1$ or $j = i + 1$, and matrix value is 0 for else. Membership function is a mathematical function that defines a fuzzy set on universe of discourse. For example U is universal set, with $U = \{x_1, x_2, x_3, \dots, x_n\}$ where x_i is a data set that granted of every data is available in the U . Therefore, the linguistic variable F_i of U can be formed as:

$$F_i = \frac{\mu_{F_i}(x_1)}{x_1} + \frac{\mu_{F_i}(x_2)}{x_2} + \frac{\mu_{F_i}(x_3)}{x_3} + \dots + \frac{\mu_{F_i}(x_n)}{x_n} \quad (7)$$

where μ_{F_i} is a membership function of fuzzy set F_i .

Fuzzy Time Series

Fuzzy time series is a prediction method using fuzzy set concept as a basic. This prediction method works by capturing patterns from past data and then projected to predict future data. The determination of interval length is an important initial stage that must be considered because it has an influence on the accuracy of prediction results in the fuzzy time series method. In the application using the determination of intervals with an average based there are 4 stages in the process. Here is the process of determining the interval (Handayani, 2015):

1. Calculate all absolute lag values with the formula:

$$\sum_{i=1}^{n-1} |(D_{i+1}) - (D_i)| \quad (8)$$

where:

- n : Total data
 D_i : The data where $i = 1, 2, \dots, n - 1$
2. The summation result of the first process is then divided by the amount of data.
 3. To determine the base of the interval based on table 2, the result of the process from number 2 is divided by 2.

Table 1. Interval Table

Range	Base
0.1 – 1.0	0.1
1.1 – 10	1
11 – 100	10
101 – 1000	100
1001 – 10000	10000

- After obtaining the base value of the interval then the range value of that base can be used as the length of the interval.

Therefore to determine the fuzzy time series model lee, there are some stages which are (Arimbawa, 2013):

- Determining the set of universes (Universe of Discourse)

At this stage, the minimum value and maximum value are searched from the actual data.

$$U = [D_{min}, D_{max}] \quad (9)$$

- Determining the class of interval fuzzy

After the length of the interval is known based on the average based above, next step calculate class of intervals for dividing datas via length of the equivalent interval.

$$Class\ of\ Interval = \frac{D_{max} - D_{min}}{Length\ Interval} \quad (10)$$

- Defining fuzzy sets on the set of universes

To convert a collection of universe that already distributed and sets of real number into sets of fuzzy sets according to interval. Fuzzy sets are generated through a $n * n$ matrix, where n is the value acquired from the universe of discourse. The membership value is defined based on the rules (Boaisha, 2010) where $1 = i = n$, which n is the total class of fuzzy interval. The value of matrix is 1 if $j = i$, matrix value is 0.5 if $j = i - 1$ or $j = i + 1$, and matrix value is 0 for else. After found the matrix then it can be formed into the fuzzy set as the following form of equation (11):

$$\begin{aligned} F_1 &= \frac{f_{11}}{x_1} + \frac{f_{12}}{x_2} + \dots + \frac{f_{1n}}{x_n} \\ F_2 &= \frac{f_{21}}{x_1} + \frac{f_{22}}{x_2} + \dots + \frac{f_{2n}}{x_n} \\ &\vdots \\ F_k &= \frac{f_{k1}}{x_1} + \frac{f_{k2}}{x_2} + \dots + \frac{f_{kn}}{x_n} \end{aligned}$$

- Fuzzification of historical data

That uses past data to calculate the membership value of each fuzzy set, which ranges from 0 to 1.

- Determining FLR (Fuzzy Logical Relationship)

Example $P(i) = F_i$ and $P(i + 1) = F_j$. Relationship between two observations in order, $P(i)$ and $P(i + 1)$ become $P(i) \rightarrow P(i + 1)$, named after fuzzy logic relations, annotated by $F_i \rightarrow F_j$, where F_i namely LHS (Left Hand Side) or month of n and F_j named with RHS (Right Hand Side) or month of $n + 1$.

- Determining FLRG (Fuzzy Logical Relationship Group)

The value of each obtained relationship will be aggregated, or FLRG (Fuzzy Logical Relationship Group). For this order of formation of FLRG utilize Lee models, so that:

All Fuzzy logic Relationships are grouped or fuzzy logic relationship groups (FLRG) into interconnected groups. For example $(F_i) : F_i \rightarrow F_{j1}, F_i \rightarrow F_{j1}$ and $F_i \rightarrow F_{j2}$. From 3 fuzzy logic relationship can be grouped Lee will generate $F_i \rightarrow F_{j1}, F_i \rightarrow F_{j1}$ and $F_i \rightarrow F_{j2}$, according to Lee $F_i \rightarrow F_{j1}$, $F_i \rightarrow F_{j1}$ can affect the predicted value then the value must be calculated.

7. Defuzzification

Outcome of fuzzy is turning to a real number according to function of membership at this point in order to calculate the prediction results. For example the result of FLRG generates $F_i \Rightarrow F, F_{j1}, F_{j1}, F_{j2}, F_{j2}, \dots, F_{jp}$ where F_i is Current State or month of n , and $F_{j1}, F_{j1}, F_{j1}, F_{j2}, F_{j2}, \dots, F_{jp}$ as Next State or month of $n + 1$. Where $F_{j1}, F_{j1}, F_{j1}, F_{j2}, F_{j2}, \dots, F_{jp}$ is the set of fuzzy & the function of membership's max-value $F_{j1}, F_{j1}, F_{j1}, F_{j2}, F_{j2}, \dots, F_{jp}$ is between interval $x_{j1}, x_{j2}, x_{j3}, \dots, x_{jp}$ and $m_{j1}, m_{j2}, m_{j3}, \dots, m_{jp}$, hence the prediction result P_{t+1} is:

$$P_{t+1} = \frac{m_{j1} + m_{j2} + \dots + m_{jp}}{p} \quad (12)$$

where p is total of mid-value and to calculate the mid-value (m_i) can be formed as:

$$m_i = \frac{(\text{upper limit } i + \text{lower limit } i)}{2} \quad (13)$$

Measurement of Prediction Effectiveness

Basically, the examination of the prediction results is carried out by means of the prediction results compared with the data that exists in reality. The method of measuring the accuracy of the model using the Average Forecasting Error Rate (AFER) value has been carried out (Widi, 2018). The prediction method can be said well if the result of AFER is small. The smaller AFER the better method. Calculations with the AFER method can be written down with:

$$AFER = \sum_{i=1}^n \frac{|F_i - P_i| / F_i}{n} \times 100\% \quad (14)$$

where:

F_i : The value of actual data on i -th data

P_i : The value of the predicted result for the i -th data

i : i -th data

n : Total data

Insurance Premiums

To calculate the insurance premiums need the strike price (K) using the predicted result of the dry paddy harvested price and it is used as the indemnity limit. Thus, in order for farmer to claim the insurance is assumed the farmer satisfied the requirement of this payoff (PO) function:

$$PO = \begin{cases} P, & S_T < K \\ 0, & \text{else} \end{cases} \quad (15)$$

where the farmer will get the P is indemnity value to cover the loss is the price of paddy at farmer level S_T needs to be lower than the predicted price K as the strike price and 0 elsewhere.

Then, in determining the crop insurance premium, a formula that will be used is using the Black Scholes model with the European cash or nothing put option approach. Because the indemnity value can only be claim at time T , and the cash or nothing put option approach means the farmer will get the straight cash P (money not asset) as the payoff function of cash or nothing. Meanwhile put option means the farmer have the right to use the put option if the price of paddy at time T is lower than the K or it can be called that the farmer suffered a loss. So that, the mathematical formula will use as the formula for cash or nothing put option:

$$Premi = P e^{-rT} N(-d_2) \quad (16)$$

where:

P : Indemnity value

r : Risk free rate

T : Time to maturity (in year)

$N(-d_2)$: Cumulative normal distribution function of $-d_2$

$$d_1 = \frac{\ln\left(\frac{S_T}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T} \quad (17)$$

where:

$N(-d_2)$: Cumulative normal distribution function of $-d_2$

S_T : Prediction paddy price at time T

K : Indemnity limit
 r : Risk free rate
 T : Time to maturity (in year)
 σ : Volatility

METHODS

Type of Research

The data used in this study is secondary data, where the author take from BPS that have provided data on the price of harvested dry paddy at the farmer level.

Research Steps

The steps that will be taken in this research are as follows :

The research analysis step is explained through the flowchart based on Figure 1, and mentioned through the following steps:

1. Input the data
2. Determining length of intervals using average based
3. Determining the Universe of Discourse
4. Determining the Class of Fuzzy Interval
5. Determining fuzzy set through matrix
6. Determining Fuzzy Logical Relationship (FLR)
7. Determining Fuzzy Logical Relationship Group (FLRG) Model Lee
8. Defuzzification
9. Extract predicted result data and calculating Average Forecasting Error Rate (AFER) value
10. Determining the return paddy price value
11. Determining the mean of return value and standard deviation
12. Determining volatility
13. Calculate the insurance premium using Black Scholes

RESULT AND DISCUSSION

Input Data

In this study, the data will be used is dry paddy harvested from January 2016 – December 2021. This data consists of 72 data from BPS.

Table 2. Dry Paddy Harvested

Month	Price (Rp)
Jan-16	5205.73
Feb-16	5211.16
Mar-16	4702.51
Apr-16	4262.42
May-16	4440.13
Jun-16	4501.12
Jul-16	4375.5
Aug-16	4479.51
Sep-16	4537.13
Oct-16	4555.26
Nov-16	4513.86
Dec-16	4622.88
Jan-17	4753.7

Month	Price (Rp)
Feb-17	4639.19
Mar-17	4373.04
...	...
...	...
Aug-21	4448.05
Sep-21	4548.23
Oct-21	4608.44
Nov-21	4650.34
Dec-21	4773.08

Determine the Length of Interval

To determine the length of interval using average based, there are 4 steps:

1. Calculate the absolute lag:

Table 3. Absolut Lag

Month	Price (Rp)	Absolute lag
		$ (D_{i+1}) - (D_i) $
Jan-16	5205.73	5.43
Feb-16	5211.16	508.65
Mar-16	4702.51	440.09
Apr-16	4262.42	177.71
May-16	4440.13	60.99
Jun-16	4501.12	125.62
Jul-16	4375.5	104.01
Aug-16	4479.51	57.62
Sep-16	4537.13	18.13
Oct-16	4555.26	41.4
Nov-16	4513.86	109.02
Dec-16	4622.88	130.82
Jan-17	4753.7	114.51
Feb-17	4639.19	266.15
Mar-17	4373.04	65.14
Apr-17	4307.9	176.68
May-17	4484.58	43.49
...
Jul-21	4310.72	137.33
Aug-21	4448.05	100.18
Sep-21	4548.23	60.21
Oct-21	4608.44	41.9
Nov-21	4650.34	122.74
Dec-21	4773.08	-
Total		9978.69

2. The summation result of the first process is then divided by the amount of data. It is $9978.69/71=140.5$
3. To determine the base of the interval, the result of the process from number 2 is divided by 2. It is $140.54/2=70$. Therefore based on table 2, it includes to the base 10 in the range 11-100
4. Then, it found that the length of interval is 70.

Determine Universe of Discourse (U)

From the data of dry paddy harvested in Table 2 we can find the maximum and minimum value, 5415.16 and 4262.42 respectively. Therefore, the value of $U = [D_{min}, D_{max}] = [4262.42, 5415.16]$.

Determine the Class of Fuzzy Interval

To determine the class of fuzzy interval, we can refer to the formula $Class\ of\ Interval = \frac{D_{max} - D_{min}}{Length\ Interval}$ and we found the length of interval is 70. Therefore, the calculation for the class of interval will be:

$$Class\ of\ Interval = \frac{5415.16 - 4262.42}{70} = 17. \text{ Hence, the data will be distributed into 17 they are } U_1, U_2, U_3, \dots, U_{17}$$

$$\begin{aligned} U_1 &= [4262.42, 4332.69] & U_{10} &= [4894.87, 4965.14] \\ U_2 &= [4332.69, 4402.96] & U_{11} &= [4965.14, 5035.42] \\ U_3 &= [4402.96, 4473.24] & U_{12} &= [5035.42, 5105.69] \\ U_4 &= [4473.24, 4543.51] & U_{13} &= [5105.69, 5175.96] \\ U_5 &= [4543.51, 4613.78] & U_{14} &= [5175.96, 5246.23] \\ U_6 &= [4613.78, 4684.05] & U_{15} &= [5246.23, 5316.51] \\ U_7 &= [4684.05, 4754.33] & U_{16} &= [5316.51, 5386.78] \\ U_8 &= [4754.33, 4824.60] & U_{17} &= [5386.78, 5457.05] \\ U_9 &= [4824.60, 4894.87] \end{aligned}$$

Determine Fuzzy Set

To determine fuzzy set, it is provided from the membership of fuzzy set F_i in between 0, 0.5, 1 where $1 = i = n$, which n is the total class of fuzzy interval. The value of matrix is 1 if $j = i$, matrix value is 0.5 if $j = i - 1$ or $j = i + 1$, and matrix value is 0 for else. Therefore, the table of matrix form is:

Table 3. Matrix Fuzzy Set

F_{ij}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0.5	1	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0.5	1	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0.5	1	0.5	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0.5	1	0.5	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0.5	1	0.5	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0.5	1	0.5	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0.5	1	0.5	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0.5	1	0.5	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0.5	1	0.5	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0.5	1	0.5	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0.5	1	0.5	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0.5	1	0.5	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0.5	1	0.5	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	1	0.5	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	1	0.5
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	1

From the table above, the matrix generates fuzzy set as follows:

$$\begin{aligned} F_1 &= \frac{1}{x_1} + \frac{0.5}{x_2} + \frac{0}{x_3} + \frac{0}{x_4} + \frac{0}{x_5} + \frac{0}{x_6} + \frac{0}{x_7} + \frac{0}{x_8} + \frac{0}{x_9} + \frac{0}{x_{10}} + \frac{0}{x_{11}} + \frac{0}{x_{12}} + \frac{0}{x_{13}} + \frac{0}{x_{14}} + \frac{0}{x_{15}} + \frac{0}{x_{16}} + \frac{0}{x_{17}} \\ F_2 &= \frac{0.5}{x_1} + \frac{1}{x_2} + \frac{0.5}{x_3} + \frac{0}{x_4} + \frac{0}{x_5} + \frac{0}{x_6} + \frac{0}{x_7} + \frac{0}{x_8} + \frac{0}{x_9} + \frac{0}{x_{10}} + \frac{0}{x_{11}} + \frac{0}{x_{12}} + \frac{0}{x_{13}} + \frac{0}{x_{14}} + \frac{0}{x_{15}} + \frac{0}{x_{16}} + \frac{0}{x_{17}} \\ F_3 &= \frac{0}{x_1} + \frac{0.5}{x_2} + \frac{1}{x_3} + \frac{0.5}{x_4} + \frac{0}{x_5} + \frac{0}{x_6} + \frac{0}{x_7} + \frac{0}{x_8} + \frac{0}{x_9} + \frac{0}{x_{10}} + \frac{0}{x_{11}} + \frac{0}{x_{12}} + \frac{0}{x_{13}} + \frac{0}{x_{14}} + \frac{0}{x_{15}} + \frac{0}{x_{16}} + \frac{0}{x_{17}} \\ F_4 &= \frac{0}{x_1} + \frac{0}{x_2} + \frac{0.5}{x_3} + \frac{1}{x_4} + \frac{0.5}{x_5} + \frac{0}{x_6} + \frac{0}{x_7} + \frac{0}{x_8} + \frac{0}{x_9} + \frac{0}{x_{10}} + \frac{0}{x_{11}} + \frac{0}{x_{12}} + \frac{0}{x_{13}} + \frac{0}{x_{14}} + \frac{0}{x_{15}} + \frac{0}{x_{16}} + \frac{0}{x_{17}} \\ F_5 &= \frac{0}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0.5}{x_4} + \frac{1}{x_5} + \frac{0.5}{x_6} + \frac{0}{x_7} + \frac{0}{x_8} + \frac{0}{x_9} + \frac{0}{x_{10}} + \frac{0}{x_{11}} + \frac{0}{x_{12}} + \frac{0}{x_{13}} + \frac{0}{x_{14}} + \frac{0}{x_{15}} + \frac{0}{x_{16}} + \frac{0}{x_{17}} \\ F_6 &= \frac{0}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0}{x_4} + \frac{0.5}{x_5} + \frac{1}{x_6} + \frac{0.5}{x_7} + \frac{0}{x_8} + \frac{0}{x_9} + \frac{0}{x_{10}} + \frac{0}{x_{11}} + \frac{0}{x_{12}} + \frac{0}{x_{13}} + \frac{0}{x_{14}} + \frac{0}{x_{15}} + \frac{0}{x_{16}} + \frac{0}{x_{17}} \\ F_7 &= \frac{0}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0}{x_4} + \frac{0}{x_5} + \frac{0.5}{x_6} + \frac{1}{x_7} + \frac{0.5}{x_8} + \frac{0}{x_9} + \frac{0}{x_{10}} + \frac{0}{x_{11}} + \frac{0}{x_{12}} + \frac{0}{x_{13}} + \frac{0}{x_{14}} + \frac{0}{x_{15}} + \frac{0}{x_{16}} + \frac{0}{x_{17}} \end{aligned}$$

$$\begin{aligned}
F_8 &= \frac{0}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0}{x_4} + \frac{0}{x_5} + \frac{0}{x_6} + \frac{0.5}{x_7} + \frac{1}{x_8} + \frac{0.5}{x_9} + \frac{0}{x_{10}} + \frac{0}{x_{11}} + \frac{0}{x_{12}} + \frac{0}{x_{13}} + \frac{0}{x_{14}} + \frac{0}{x_{15}} + \frac{0}{x_{16}} + \frac{0}{x_{17}} \\
F_9 &= \frac{0}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0}{x_4} + \frac{0}{x_5} + \frac{0}{x_6} + \frac{0}{x_7} + \frac{0.5}{x_8} + \frac{1}{x_9} + \frac{0.5}{x_{10}} + \frac{0}{x_{11}} + \frac{0}{x_{12}} + \frac{0}{x_{13}} + \frac{0}{x_{14}} + \frac{0}{x_{15}} + \frac{0}{x_{16}} + \frac{0}{x_{17}} \\
F_{10} &= \frac{0}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0}{x_4} + \frac{0}{x_5} + \frac{0}{x_6} + \frac{0}{x_7} + \frac{0}{x_8} + \frac{0}{x_9} + \frac{0.5}{x_{10}} + \frac{1}{x_{11}} + \frac{0.5}{x_{12}} + \frac{0}{x_{13}} + \frac{0}{x_{14}} + \frac{0}{x_{15}} + \frac{0}{x_{16}} + \frac{0}{x_{17}} \\
F_{11} &= \frac{0}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0}{x_4} + \frac{0}{x_5} + \frac{0}{x_6} + \frac{0}{x_7} + \frac{0}{x_8} + \frac{0}{x_9} + \frac{0}{x_{10}} + \frac{0.5}{x_{11}} + \frac{1}{x_{12}} + \frac{0.5}{x_{13}} + \frac{0}{x_{14}} + \frac{0}{x_{15}} + \frac{0}{x_{16}} + \frac{0}{x_{17}} \\
F_{12} &= \frac{0}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0}{x_4} + \frac{0}{x_5} + \frac{0}{x_6} + \frac{0}{x_7} + \frac{0}{x_8} + \frac{0}{x_9} + \frac{0}{x_{10}} + \frac{0}{x_{11}} + \frac{0.5}{x_{12}} + \frac{1}{x_{13}} + \frac{0.5}{x_{14}} + \frac{0}{x_{15}} + \frac{0}{x_{16}} + \frac{0}{x_{17}} \\
F_{13} &= \frac{0}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0}{x_4} + \frac{0}{x_5} + \frac{0}{x_6} + \frac{0}{x_7} + \frac{0}{x_8} + \frac{0}{x_9} + \frac{0}{x_{10}} + \frac{0}{x_{11}} + \frac{0}{x_{12}} + \frac{0.5}{x_{13}} + \frac{1}{x_{14}} + \frac{0.5}{x_{15}} + \frac{0}{x_{16}} + \frac{0}{x_{17}} \\
F_{14} &= \frac{0}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0}{x_4} + \frac{0}{x_5} + \frac{0}{x_6} + \frac{0}{x_7} + \frac{0}{x_8} + \frac{0}{x_9} + \frac{0}{x_{10}} + \frac{0}{x_{11}} + \frac{0}{x_{12}} + \frac{0.5}{x_{13}} + \frac{1}{x_{14}} + \frac{0.5}{x_{15}} + \frac{0}{x_{16}} + \frac{0}{x_{17}} \\
F_{15} &= \frac{0}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0}{x_4} + \frac{0}{x_5} + \frac{0}{x_6} + \frac{0}{x_7} + \frac{0}{x_8} + \frac{0}{x_9} + \frac{0}{x_{10}} + \frac{0}{x_{11}} + \frac{0}{x_{12}} + \frac{0}{x_{13}} + \frac{0.5}{x_{14}} + \frac{1}{x_{15}} + \frac{0.5}{x_{16}} + \frac{0}{x_{17}} \\
F_{16} &= \frac{0}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0}{x_4} + \frac{0}{x_5} + \frac{0}{x_6} + \frac{0}{x_7} + \frac{0}{x_8} + \frac{0}{x_9} + \frac{0}{x_{10}} + \frac{0}{x_{11}} + \frac{0}{x_{12}} + \frac{0}{x_{13}} + \frac{0}{x_{14}} + \frac{0.5}{x_{15}} + \frac{1}{x_{16}} + \frac{0.5}{x_{17}} \\
F_{17} &= \frac{0}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0}{x_4} + \frac{0}{x_5} + \frac{0}{x_6} + \frac{0}{x_7} + \frac{0}{x_8} + \frac{0}{x_9} + \frac{0}{x_{10}} + \frac{0}{x_{11}} + \frac{0}{x_{12}} + \frac{0}{x_{13}} + \frac{0}{x_{14}} + \frac{0}{x_{15}} + \frac{0.5}{x_{16}} + \frac{1}{x_{17}}
\end{aligned}$$

Determine FLR (Fuzzy Logical Relationship)

Before determine fuzzy logical relationship, we need to find the fuzzification first. Based on the data of dry paddy harvested price, on January 2016 the fuzzification is F_{14} , because the price is 5205.73 where it includes in between the value of interval of $U_{14} = [5175.96, 5246.23]$, and so on until December 2021. Therefore, we can determine the FLR depict on table below:

Table 4. Fuzzy Logical Relationship

Month	Price (Rp)	Fuzzification			FLR
Jan-16	5205.73	F14	NA	⊗	F14
Feb-16	5211.16	F14	F14	⊗	F14
Mar-16	4702.51	F7	F14	⊗	F7
Apr-16	4262.42	F1	F7	⊗	F1
May-16	4440.13	F3	F1	⊗	F3
Jun-16	4501.12	F4	F3	⊗	F4
Jul-16	4375.5	F2	F4	⊗	F2
Aug-16	4479.51	F4	F2	⊗	F4
Sep-16	4537.13	F4	F4	⊗	F4
Oct-16	4555.26	F5	F4	⊗	F5
Nov-16	4513.86	F4	F5	⊗	F4
Dec-16	4622.88	F6	F4	⊗	F6
Jan-17	4753.7	F7	F6	⊗	F7
Feb-17	4639.19	F6	F7	⊗	F6
...
Aug-21	4448.05	F3	F1	⊗	F3
Sep-21	4548.23	F5	F3	⊗	F5
Oct-21	4608.44	F5	F5	⊗	F5
Nov-21	4650.34	F6	F5	⊗	F6
Dec-21	4773.08	F8	F6	⊗	F8

Determine FLRG (Fuzzy Logical Relationship Group)

According to the FLR result, next will be formed the FLRG Lee Model that can be seen on the table below:

Table 5. Fuzzy Logical Relationship Group

FLRG		
F1	⊙	F2, F3, F3, F4
F2	⊙	F1, F1, F2, F4, F5, F5
F3	⊙	F4, F5
F4	⊙	F2, F4, F4, F4, F4, F5, F6, F6
F5	⊙	F1, F2, F4, F5, F5, F6, F6, F6. F6
F6	⊙	F2, F6, F7, F7, F8, F8, F8, F8
F7	⊙	F1, F6, F8, F8
F8	⊙	F2, F5, F7, F8, F9, F9, F9, F10, F10
F9	⊙	F8, F10, F11
F10	⊙	F5, F8, F11, F13
F11	⊙	F12, F17
F12	⊙	F14
F13	⊙	F5, F14
F14	⊙	F7, F8, F10, F14, F15, F16
F15	⊙	F14
F16	⊙	F13
F17	⊙	F14

Defuzzification

To find the defuzzification, we need to calculate the middle value first with the formula is $m_i = \frac{(\text{upper limit } i + \text{lower limit } i)}{2}$ for $i = 17$ of class fuzzy interval in U (Universe of Discourse). Then, the table as follows:

Table 6. Middle Value

Universe of Discourse				Middle Value
U1	4262.42	,	4332.69	m1 4297.56
U2	4332.69	,	4402.96	m2 4367.83
U3	4402.96	,	4473.24	m3 4438.10
U4	4473.24	,	4543.51	m4 4508.37
U5	4543.51	,	4613.78	m5 4578.65
U6	4613.78	,	4684.05	m6 4648.92
U7	4684.05	,	4754.33	m7 4719.19
U8	4754.33	,	4824.60	m8 4789.46
U9	4824.60	,	4894.87	m9 4859.74
U10	4894.87	,	4965.14	m10 4930.01
U11	4965.14	,	5035.42	m11 5000.28
U12	5035.42	,	5105.69	m12 5070.55
U13	5105.69	,	5175.96	m13 5140.83
U14	5175.96	,	5246.23	m14 5211.10
U15	5246.23	,	5316.51	m15 5281.37
U16	5316.51	,	5386.78	m16 5351.64
U17	5386.78	,	5457.05	m17 5421.92

Therefore, to determine the prediction result is can be seen as the table below:

Table 7. Calculation of Prediction Result

FLRG	Calculation	Prediction Result
F1 ⊗ F2, F3, F3, F4	$\frac{m2+m3+m3+m4}{4}$	4438.101162
F2 ⊗ F1, F1, F2, F4, F5, F5	$\frac{m1+m1+m2+m4+m5+m5}{6}$	4438.101162
F3 ⊗ F4, F5	$\frac{m4+m5}{2}$	4543.509859
F4 ⊗ F2, F4, F4, F4, F4, F5, F6, F6	$\frac{m2+m4+m4+m4+m4+m5+m6+m6}{8}$	4534.725801
F5 ⊗ F1, F2, F4, F5, F5, F6, F6, F6, F6	$\frac{m1+m2+m4+m5+m5+m6+m6+m6+m6}{9}$	4547.413885
F6 ⊗ F2, F6, F7, F7, F8, F8, F8, F8	$\frac{m2+m6+m7+m7+m8+m8+m8+m8}{8}$	4701.622905
F7 ⊗ F1, F6, F8, F8	$\frac{m1+m6+m8+m8}{4}$	4631.35044
F8 ⊗ F2, F5, F7, F8, F9, F9, F9, F10, F10	$\frac{m2+m5+m7+m8+m9+m9+m9+m10+m10}{9}$	4766.039331
F9 ⊗ F8, F10, F11	$\frac{m8+m10+m11}{3}$	4906.584261
F10 ⊗ F5, F8, F11, F13	$\frac{m5+m8+m11+m13}{4}$	4877.304067
F11 ⊗ F12, F17	$\frac{m12+m17}{2}$	5246.234507
F12 ⊗ F14	$m14$	5211.098275
F13 ⊗ F5, F14	$\frac{m5+m14}{2}$	4894.872183
F14 ⊗ F7, F8, F10, F14, F15, F16	$\frac{m7+m8+m10+m14+m15+m16}{6}$	3981.979852
F15 ⊗ F14	$m14$	5211.098275
F16 ⊗ F13	$m13$	5140.82581
F17 ⊗ F14	$m14$	5211.098275

Extract Predicted Result Data & Calculate AFER Value

The value of AFER (Average Forecasting Error Rate) can be seen on table below:

Table 8. Prediction Result & AFER Calculation

Month	Actual Data	Prediction Result	AFER
Jan-16	5205.73	NA	NA
Feb-16	5211.16	3981.979852	0.3261564%
Mar-16	4702.51	3981.979852	0.2128090%
Apr-16	4262.42	4631.35044	0.1202142%
May-16	4440.13	4438.101162	0.0006346%
Jun-16	4501.12	4543.509859	0.0130800%
Jul-16	4375.5	4534.725801	0.0505421%
Aug-16	4479.51	4438.101162	0.0128390%
Sep-16	4537.13	4534.725801	0.0007360%
Oct-16	4555.26	4534.725801	0.0062608%
Nov-16	4513.86	4547.413885	0.0103243%
Dec-16	4622.88	4534.725801	0.0264849%
Jan-17	4753.7	4701.622905	0.0152154%
Feb-17	4639.19	4631.35044	0.0023470%
Mar-17	4373.04	4701.622905	0.1043588%
Apr-17	4307.9	4438.101162	0.0419775%
May-17	4484.58	4438.101162	0.0143946%
...
Apr-21	4274.9	4438.101162	0.0530231%
May-21	4397.79	4438.101162	0.0127309%
Jun-21	4545.56	4438.101162	0.0328339%
Jul-21	4310.72	4547.413885	0.0762614%
Aug-21	4448.05	4438.101162	0.0031065%
Sep-21	4548.23	4543.509859	0.0014414%
Oct-21	4608.44	4547.413885	0.0183920%
Nov-21	4650.34	4547.413885	0.0307403%
Dec-21	4773.08	4701.622905	0.0207929%
		Average AFER	0.0533919%

Then, we found the average of AFER value which is 0.05%. That means the error rate is small. Therefore, the data both actual and predicted result for the next 3 periods are using the last 3 predicted data on the table 10, they are 4547.41, 4547.41, and 4701.62. After that, the last predicted result which is 4701.62 will be used as K for determining the premium insurance.

Determine Return Paddy Price Value

To find the return value of the price dry paddy harvested. The result is in the table below:

Table 9. Return Value

Month	Price/kg	Return value (R_i)	$R_i - \bar{R}_t$	$(R_i - \bar{R}_t)^2$
Jan-16	5205.73	NA	NA	NA
Feb-16	5211.16	0.001043	0.002264624	5.12852E-06
Mar-16	4702.51	-0.10271	-0.101483984	0.010298999
Apr-16	4262.42	-0.09826	-0.097037248	0.009416228
May-16	4440.13	0.040847	0.042068668	0.001769773
Jun-16	4501.12	0.013643	0.014864686	0.000220959
Jul-16	4375.5	-0.02831	-0.027083369	0.000733509
Aug-16	4479.51	0.023493	0.024714953	0.000610829
Sep-16	4537.13	0.012781	0.014003075	0.000196086
Oct-16	4555.26	0.003988	0.005210042	2.71445E-05
Nov-16	4513.86	-0.00913	-0.007907859	6.25342E-05
Dec-16	4622.88	0.023865	0.025087311	0.000629373
Jan-17	4753.7	0.027905	0.029127461	0.000848409
Feb-17	4639.19	-0.02438	-0.023161394	0.00053645
Mar-17	4373.04	-0.05908	-0.057859276	0.003347696
...
...
...
Aug-21	4448.05	0.031361	0.032582941	0.001061648
Sep-21	4548.23	0.022272	0.023494435	0.000551988
Oct-21	4608.44	0.013151	0.014373345	0.000206593
Nov-21	4650.34	0.009051	0.010273017	0.000105535
Dec-21	4773.08	0.026051	0.02727355	0.000743847
	Total	-0.08677	Total	0.103528498

Determine Mean and Standard Deviation of Return Value

Then, to find the mean which notated with $\bar{R}_t = \frac{\text{total of return value}}{n} = -\frac{0.086768143}{71} = -0.001222087$. Meanwhile the standard deviation is $s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (R_i - \bar{R}_t)^2} = \sqrt{\frac{1}{71-1} (0.103528498)} = 0.03845749$.

Determine Volatility To Calculate Crop Insurance Premium

Volatility calculation is $\sigma = \frac{s}{\sqrt{\frac{1}{k}}} = \frac{0.03845749}{\sqrt{\frac{1}{12}}} = 0.133220654$. Therefore, to determine the crop insurance premium with the condition of $K > S_T$ is as follows as below:

1. Find $N(-d_1)$ and $N(-d_2)$

$$d_1 = \frac{\ln\left(\frac{S_T}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{4653}{4701.62}\right) + \left(0.07 + \frac{0.133220654^2}{2}\right)0.25}{0.133220654\sqrt{0.25}} = 0.139961671$$

$N(d_1) = 0.555654863$ after calculated using Microsoft Excel or it can be refer to the Z score table

$$N(-d_1) = 1 - 0.555654863 = 0.444345137.$$

Then, find the $N(-d_2)$:

$$d_2 = d_1 - \sigma\sqrt{T} = 0.139961671 - 0.133220654\sqrt{0.25} = 0.073351344$$

$$N(d_2) = 0.529236732 \text{ same way as } N(d_1).$$

$$N(-d_2) = 1 - 0.529236732 = 0.470763268.$$

2. Calculate the premiums

$$Premi = Pe^{-rT}N(-d_2) = 6,000,000 * e^{-0.07*0.25} * 0.470763268$$

$Premi = 2,775,579$. Therefore, the premi for the farmers if the indemnity value is 6,000,000 and time is 0.25 (3 months) is 2,775,579. Meanwhile, based on *Asuransi Usaha Tani Padi* (AUTP) only pay 36,000. Therefore, it is too expensive as an insurance premiums.

CONCLUSION

Based on the results of the analysis and discussion, the following conclusions can be drawn: According to the analysis and result on the previous chapter, there are some important things that can be sum up are:

1. The prediction results value of paddy prices using the Fuzzy Time Series Model Lee method for the next three periods such as January 2022, February 2022, and March 2022 are 4547.41, 4547.41, and 4701.62.
2. The accuracy level of prediction results value using Average Forecasting Error Rate (AFER) is 0.05%, where it is a small value means that the accuracy level of prediction result using Fuzzy Time Series Model Lee is good.
3. The crop insurance premiums based on the predicted result of paddy prices using Black Scholes cash or nothing put option approach method is 2,775,579. That means, this method of calculations is pricey to be the insurance premiums.

There are also the suggestions given by the writer in this study, some of the things can be conclude in:

1. Through this study result, for future study to develop analysis study using another method.
2. To determine the insurance premiums using another method and another based that will generate the result more effective and cheaper.
3. For future study using mathematical software or other tools instead of using manually calculator.

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Conflicts of Interest

The authors declare no conflict of interest.

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