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Metric Dimension of the Pinwheel Subdivision Graph $K_1 + mK_3$

Vetty Sugiarty[1*], Amrullah[1], Nurul Hikmah[1]

[1] Mathematics Education Department, Faculty of Teacher Training and Education, University of Mataram, Indonesia

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Corresponding Author:
Vetty Sugiarty
sugiartyvetty@gmail.com

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Abstract: Metric dimension is an important concept in graph theory that is widely used in various fields, including navigation, network localization, and network design. The concept of metric dimension is the concept of determining the least marker vertex so that each vertex in the graph is distinguished from each other. The purpose of this research is to determine the metric dimension of the pinwheel subdivision graph $K_1 + mK_3$. The type of research used is pure research. By using graph structure and vertex distance analysis, this paper shows the value of the metric dimension of the subdivision graph $S(G, e)$, specifically on the pinwheel graph $G = K_1 + mK_3$ for $2 \le m \le 4$. The results show that the metric dimension of the pinwheel subdivision graph $K_1 + mK_3$ is one less than the metric dimension of the pinwheel graph before subdivision, $dim(S(G, e)) = dim(G) - 1$.

Keywords: Metric Dimension; Pinwheel Graph $K_1 + mK_3$; Subdividing Set.

INTRODUCTION

Graph theory is a branch of mathematics that is very useful in helping to solve problems in real life. By presenting real-life problems in graph form, a problem will be easy to understand and easier to find a solution (Haspika, Hasmawati, & Aris, 2023). One of the themes that become research studies in graph theory is metric dimension. According to (Chartrand, Eroh, Johnson, & Oellerman, 2000), the concept of metric dimension originated from the idea of distinguishing sets and minimum distinguishing sets introduced by (Slater, 1975). Slater introduced the concept of locating set, which defines location number as the smallest cardinality of the set of distinguishing vertices in graph G. It was later developed by (Harary & Melter, 1976). It states that the metric dimension of a graph G is defined as the smallest cardinality of the currently known metric bases.

In the concept of metric dimension, suppose u and v are two vertices in a connected graph G, then the distance from u to v is the length of the shortest path between u and v in G denoted by $d(u, v)$. For an ordered set $W = (w_1, w_2, \dots, w_k)$ of vertices in a graph G and vertex v in G, the representation of v with respect to W is a k-vector (k-tuple pair).

$$r(v|W) = \left(d(v, w_1), d(v, w_2), \dots, d(v, w_k)\right)$$

If the representation of v on W for every vertex v in G is different, then W is called the distinguishing set of the graph G. The distinguishing set with the smallest cardinality is called the smallest distinguishing set (metric basis), and the cardinality of the metric basis is called the metric dimension of G and is denoted by $dim(G)$ (Chartrand et al., 2000).

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An example of an application that utilizes the metric dimension in graphs is robot navigation (Franz, Scholkopf, Mallot, & Bulthoff, 1998), where a robot moves from one vertex location to another in space by minimizing the error that occurs in interpreting the coordinates obtained from the vertices. Therefore, each location node in the robot's motion field must provide unique and distinct coordinates. If location nodes are viewed as vertices and robot trajectories are viewed as edges, then the robot motion space can be represented as a graph.

The problem of metric dimension is to determine the marker vertices so that the distinct and unique coordinates have the smallest possible element. If the marker vertex element is the distance of the marker vertices that have the smallest possible element, then determining the smallest possible distinguishing set is called metric dimension (Khuller, Raghavachari, & Rosenfeld, 1996).

Research on metric dimension in graphs is a topic that is currently in high demand. This is evidenced by the number of journals and researches that discuss this study even with another type of dimension, known as partition dimension, finding partition dimension in graphs is still an open problem in graph theory. Therefore, several researchers investigated the problem (Amrullah, Azmi, Turmuzi, Baidowi, & Kurniati, 2021) according to (Amrullah, Baskoro, Uttunggadewa, & Simanjuntak, 2016) The partition dimension, $pd(G)$ of G is the smallest integer p such that G has a completion partition p.

Some about metric dimensions have been published including, "Metric Dimension of Pinwheel Graph Pattern $K_1 + mK_4$" has been published by (Riyandho, Narwen, & Efendi, 2018) the results of the study concluded that, the metric dimension of pinwheel graph pattern $K_1 + mK_4$ for $m \geq 2$ is $3m$, then "Metric Dimension of Pinwheel Graph Development Pattern $K_1 + mK_3$" has been published by (Wahyudi, Sumarno, & Suharmadi, 2011) the results of the study concluded that the metric dimension of the development of the pinwheel graph pattern $K_1 + mK_3$ with $m \geq 2$ is $2m$, then "The Metric Dimension of the Bridge Operation Result Graph of Homogeneous Caterpillar and Generalized Flower Pot" published by (Hidayanti, Amrullah, Kurniati, & Hayati, 2022). The results concluded that the metric dimension of the bridge graph of the homogeneous caterpillar graph C(m,n) and the generalized flowerpot $C_P - K_{q1,q2,\dots,qp}$ is at least two less than the value of $m(n-1) + \sum_{i=1}^{p} q_i - 2p$. The largest metric dimension of the bridge graph is equal to $m(n-1) + \sum_{i=1}^{p} q_i - 2p$.

A pinwheel graph can also be called a complete graph K_n copied as many as m copies with a point as the common center point of all copies of the complete graph (Riyandho et al., 2018). The pinwheel graph $K_1 + mK_3$ is a type of graph that consists of the complete graph $K_1$and the complete graph K_3 of m copies (Miller, Patel, Ryan, Sugeng, Slamin, & Tuga, 2005). The K_1 graph consists of one vertex, while the complete K_3 graph has m vertices and m edges, where each vertex is connected to a center vertex and the other vertices form a triangle.

The advantage of the pinwheel graph pattern $K_1 + mK_3$ is that it tends to have efficiency in data representation because it reduces the number of edges that need to be stored compared to graphs that do not have this pattern. This can certainly save storage space and speed up the data analysis process, besides providing an intuitive visual representation, allowing to quickly understand the connection pattern between vertices in the graph.

One of the methods used by researchers is graph operations, such as subdivision operations (Amrullah, Azmi, Soeprianto, & Anwar, 2018). However, from previous studies on the metric dimension of pinwheel graphs, they only discuss the metric dimension structure of the graph without a graph operation. Therefore, this research will use one of the graph operations, namely the subdivision operation. Suppose G is a randomly chosen graph, for every positive integer k and $e \in E(G)$, a subdivision of graph G, denoted by $S(G(e{:}\,k))$, is a graph obtained from G by replacing edge e with path $(k+1)$ (Amrullah, 2020). Meanwhile, according to (Yulianti, Hidayati, & Welyyanti, 2023) subdivision on a graph is performing a subdivision operation on a particular edge, by inserting a new vertex along one or more edges of the graph.

This study aims to determine the metric dimension of the pinwheel subdivision graph $K_1 + mK_3$, the pinwheel graph $K_1 + mK_3$ was chosen because no previous researchers have discussed the pinwheel graph $K_1 + mK_3$, thus the results of this study are expected to be a reference for future researchers who will examine the metric dimension of other graph subdivisions.

METHOD

The type of research used in this research is pure research. This research uses a graph structure analysis approach by paying attention to the order, distance, vertex degree, to determine the metric dimension of the graph. As for this research, the data collection technique is carried out from analyzing the design of the pinwheel graph $K_1 + mK_3$ by paying attention to the value of m. Starting with $m = 2$, subdivision is carried out on the edges of the graph. Starting with $m = 2$, subdivision is done on the edges connected to the center vertex. then the determination of the discriminating set on each subdivision e. The determination of the minimal discriminating set is done by considering the distance pattern between vertices and the degree of the largest vertex. For $m = 3\ and\ 4$, the determination of the minimal distinguishing set is done in the same way as $m = 2$.

In this research, there are several steps that can be done as follows:

1. Construction of the pinwheel graph $K_1 + mK_3$starting with $m = 2$.
2. Construct the distinguishing set of the pinwheel subdivision graph by: (a) Subdivision is done on the edge associated with the center vertex by adding vertices on that edge; (b) Retrieval of some vertices in the complete graph K_3 is included in the distinguishing set; (c) Perform several combinations of vertex retrieval on all complete graphs; (d) Find the distance of each vertex to all vertices in the distinguishing set. If the vertex is not a distinguishing set, then continue with the combination of vertices that have not been drawn before; (e) Determine the representation of each vertex of the graph with the distinguishing set; (f) Subdivision is then performed on the other edges in the complete graph K_3.
3. Determines the metric dimension of the pinwheel subdivision graph based on the correct distinguishing set. This is done by general proof on the pinwheel graph with subdivision for each edge of the graph.
4. The proof of the metric dimension result on the pinwheel subdivision graph $K_1 + mK_3$ is based on data that has been obtained on the $m = 2\ to\ = 4$ pinwheel graphs. This general proof uses the properties of the distance between vertices and the vertex location of the distinguishing set. This proof will apply to the pinwheel graph $K_1 + mK_3$ with $2 \leq m \leq 4$ in general.

RESULT AND DISCUSSION

Lemma 1

Suppose $G = K_1 + nK_3$, $D_i = \{u_{i1}, u_{i2}, u_{i3}\} \subseteq V(G)$ and W is the difference set of G then there are at least 2 vertices in $D_i \in W$.

Proof:

Suppose there is one vertex in $D_i \in W$, only $u_{11} \in D_i$. This results in vertices $u_{12}$and u_{13} being 1-distance to u_{11}, while the distances of vertices u_{12} and u_{13} to all other vertices are equal, hence $r(u_{12}|W = r(u_{13} + W)$. Contradiction.

Theorem 1

If the pinwheel graph $G = K_1 + mK_3$ and $e \in E(G)$ then $dim(S(G,e)) = 3$.

Proof:

To prove this, two possible cases will be considered

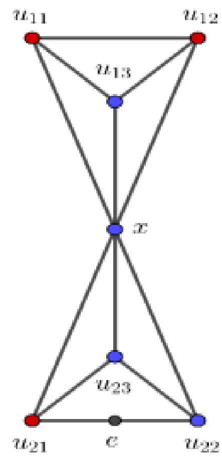

Figure 1. Pinwheel Graph $K_1 + 2K_3$

Case 1: The subdivision edge $e = u_{i1}u_{i2}$ with $i \in \{1,2\}$, is an edge that is not adjacent to the center vertex x. Without loss of generality suppose $i = 2$, Select the set $W = \{u_{11}, u_{12}, u_{21}\}$, having 3 vertices with only one member vertex u_{21} from the edge adjacent to the subdivision edge. Consider Figure 1 $e = u_{21}u_{22}$. It will then be shown that all vertices that are not members of W have different representations $\{u_{13}, x, u_{23}, u_{22}, e\}$. The five vertices have different representations which are (1,1,2), (1,1,1), (2,2,1), (2,2,2), (3,3,1) respectively.

This shows that $dim(S(G,e)) \leq 3$.

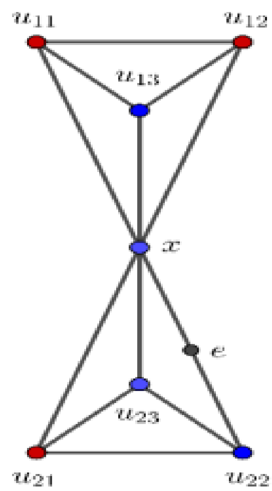

Figure 2. Pinwheel Graph $K_1 + 2K_3$

Case 2: Subdivision edge $e = u_{ij}$ with $i, j \in \{1,2\}$ is an edge adjacent to the center vertex x. Without loss of generality, suppose $u_{ij} = u_{22}$, the set $W = \{u_{11}, u_{12}, u_{21}\}$ with 3 vertices and only one member vertex that is not adjacent to the subdivision vertex e. Consider Figure 2 $e = u_{22}x$. It will then be shown that the non-member vertices of W namely $\{u_{13}, x, u_{23}, u_{22}, e\}$ have different representations. The five vertices have different representations namely (1,1,2), (1,1,1), (2,2,1), (3,3,1), (2,2,2) respectively.

This shows that $dim(S(G,e)) \leq 3$. Both cases show that for every $e \in E(G)$ $dim(S(G,e)) \leq 3$ holds.

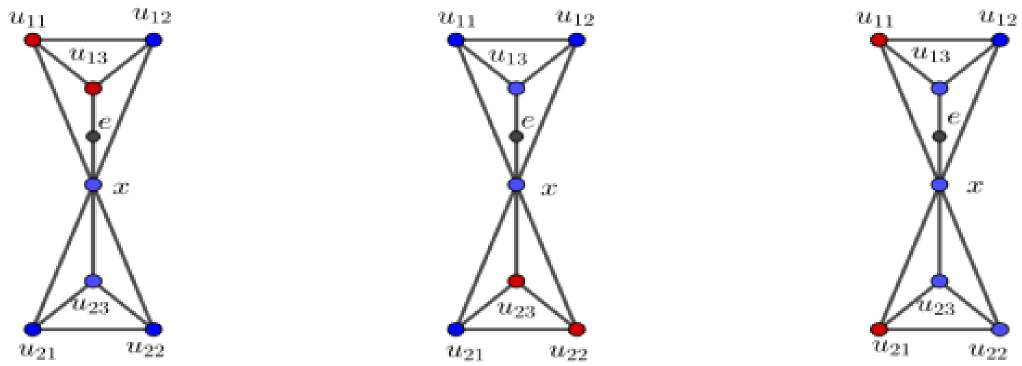

Figure 3. Pinwheel Graph $K_1 + 2K_3$

Next it will be proved that $dim(S(G, e)) \geq 3$. Suppose $dim(S(G, e)) = 2$. This means there are W subdivision sets with $|W| = 2$, suppose $W = \{z_1, z_2\}$. Suppose the subdivision edge $e = u_{13}x$ or $e = u_{12}u_{13}$ Consider Figure 3, suppose $A = \{u_{11}, u_{12}, u_{13}\}$ and $B = \{u_{21}, u_{22}, u_{23}\}$. There are three possibilities for the vertex elements in W to be in A or B. First, if $z_1, z_2 \in A$ then $r(u_{21}|W = r(u_{23}|W)$ is a contradiction. Second, if $z_1, z_2 \in B$ then $r(u_{11}|W = r(u_{12}|W)$is a contradiction. If $z_1 \in A$, $z_2 \in B$ e.g. $z_1 = u_{11}$, $z_2 = u_{21}$ then $r(u_{22}|W = r(u_{23}|W\ ) =$ (2,1) contradiction. Based on these three possibilities, it shows that $dim(S(G, e)) \geq 3$.

Therefore, based on inequalities (1) and (2), we get $dim(S(G, e)) = 3$ for any subdivision edge in $G = K_1 + 2K_3$.

Theorem 2

If the pinwheel graph $G = K_1 + 3K_3$ and $e \in E(G)$ then $dim(S(G, e)) = 5$

Proof:

This can be shown by analyzing the two alternative cases

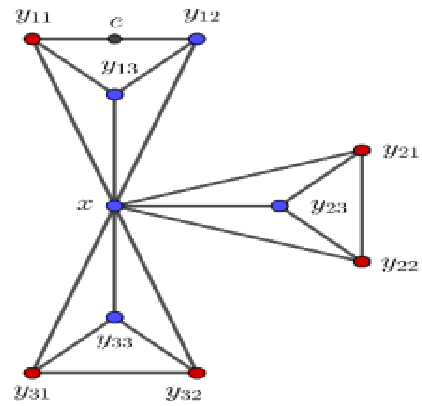

Figure 4. Pinwheel Graph $K_1 + 3K_3$

Case 1: A subdivision edge $e = y_{i1}y_{i2}$with $i \in \{1,2,3\}$, is an edge that does not share a vertex with the center vertex x. Without changing its generality, suppose $i = 1$, Select the set $W = \{y_{11}, y_{21}, y_{22}, y_{31}, y_{32}\}$ has 5 vertices with only one vertex member y_{11} from the edge that shares a vertex with the subdivision edge. Consider Figure 4 $e = y_{11}y_{12}$. Next it will be proved that each vertex not included in W has a unique representation $\{y_{12}, y_{13}, x, y_{23}, y_{33}, e\}$. The six vertices sequentially have unique representations of each other namely (2,2,2,2,2), (1,2,2,2,2), (1,1,1,1,1),
(2,1,1,2,2),(2,2,2,1,1),(1,3,3,3,3,).

This shows that $dim(S(G, e)) \leq 5$.

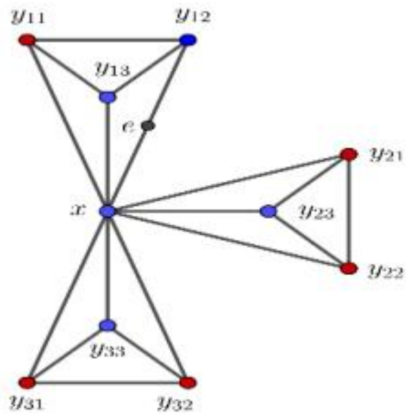

Figure 5: Pinwheel Graph $K_1 + 3K_3$

Case 2: Subdivision edge $e = y_{ij}$ with $i, j \in \{1,2,3\}$ is an edge that shares a vertex with the center vertex x. Without changing its generality, suppose $y_{ij} = y_{12}$, the set $W = \{y_{11}, y_{21}, y_{22}, y_{31}, y_{32}\}$ with 5 vertices and only one vertex that is not a member of W can share a vertex with the subdivision vertex e. Consider Figure 5 $e = y_{12}x$. Next, it will be proved that each vertex that does not belong to W has a unique representation, namely $\{y_{12}, y_{13}, x, y_{23}, y_{33}, e\}$. The six vertices sequentially have unique representations of each other namely (1,3,3,3,3), (1,2,2,2,2), (1,1,1,1,1), (2,1,1,2,2),(2,2,2,1,1),(2,2,2,2,2).

This shows that $dim(S(G, e)) \leq 5$. Both cases show that for every $e \in E(G)$ $dim(S(G, e)) \leq 5$ holds. (1)

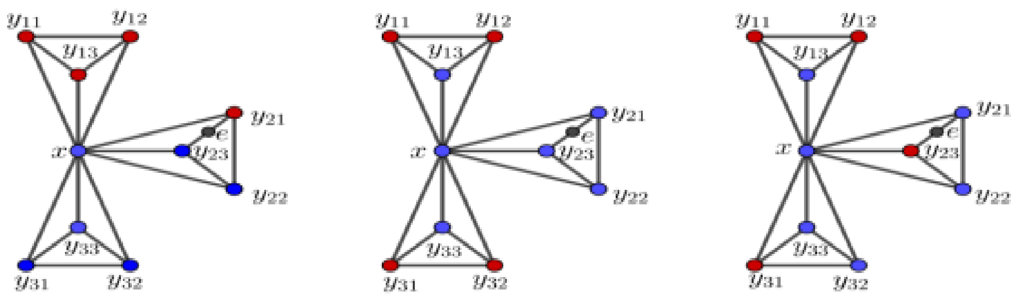

Figure 6: Pinwheel Graph $K_1 + 3K_3$

Then it will be shown that $dim(S(G, e)) \geq 5$. Suppose $dim(S(G, e)) = 4$. This shows that there is a subdivision set W with $|W| = 4$, suppose $W = z_1, z_2, z_3, z_4\}$. Suppose the subdivision edge $e = y_{23}x$ or $e = y_{21}y_{23}$. Let distinguishing set $A = \{y_{11}, y_{12}, y_{13}\}$set $B = \{y_{21}, y_{22}, y_{23}\}$set $C = \{y_{31}, y_{32}, y_{33}\}$. Since there are 4 vertices in W, there are at most 3 vertices in A, B, or C with three possible vertex elements in W. First suppose 3 vertices in A i.e. $z_1, z_2, z_3 \in A$ and $z_4 \in B$ or C, suppose $z_4 = y_{21}$ then $r(y_{31}|W) = r(y_{33}|W)$ contradiction. Second if $z_1, z_2 \in A$ and $z_3, z_4 \in B$ or C, e.g $z_3 = y_{31}$, $z_4 = y_{32}$then $r(y_{21}|W) = r(y_{22}|W)$ is a contradiction. Thirdly if $z_1, z_2 \in A$, $z_3 \in B$ and $z_4 \in C$, e.g. $z_3 = y_{23}$, $z_4 = y_{31}$ then $r(y_{32}|W) = r(y_{33}|W)$ = (2,2,2,1) contradiction.

Through these three possibilities, it can be shown that $dim(S(G, e)) \geq 5$.

Consequently, based on inequalities (1) and (2), it follows that $dim(S(G, e)) = 5$ for any subdivision edge in $G = K_1 + 3K_3$

Theorem 3

If the pinwheel graph $G = K_1 + 4K_3$ and $e \in E(G)$ then $dim(S(G, e)) = 7$

Proof:

The statement can be proven by evaluating two possible cases

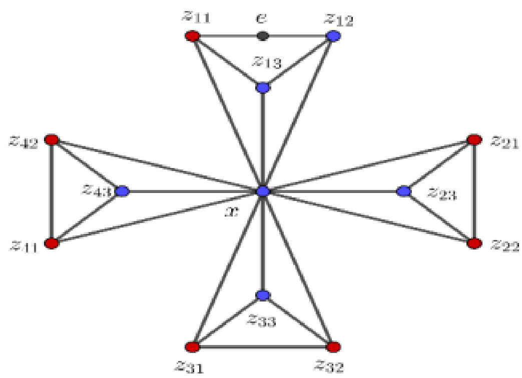

Figure 7. Pinwheel Graph $K_1 + 4K_3$

Case 1: A subdivision edge $e = z_{i1}z_{i2}$ with $i \in \{1,2,3,4\}$, is an edge that is not connected to the center vertex x. Without affecting its main characteristic, suppose $i = 1$, Select the set $W = \{z_{11}, z_{21}, z_{22}, z_{31}, z_{32}, z_{41}, z_{42}\}$ has 7 vertices with only one member vertex z_{11} of the edge connected to the subdivision edge. Consider Figure 7 $e = z_{11}z_{12}$. Next it will be shown that each vertex outside the set W has a unique representation $\{z_{12}, z_{13}, x, z_{23}, z_{33}, z_{43}, e\}$. The seven vertices have sequentially distinctive representations of each other namely (2,2,2,2,2,2,2,2), (1,2,2,2,2,2,2,2), (1,1,1,1,1,1,1),(2,1,1,2,2,2,2), (2,2,2,1,1,2,2),(2,2,2,2,2,1,1), (1,3,3,3,3,3,3)

This shows that $dim(S(G,e)) \leq 7$.

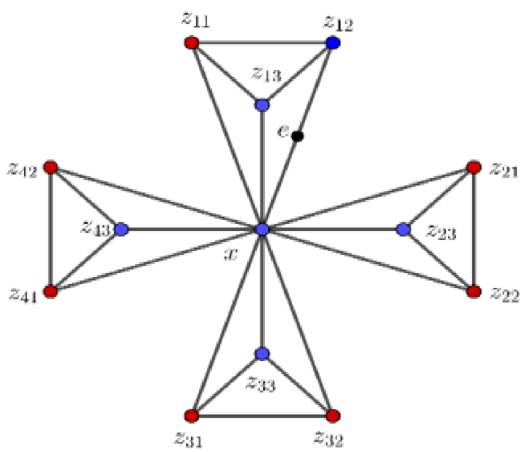

Figure 8. Pinwheel Graph $K_1 + 4K_3$

Case 2: Subdivision edge $e = z_{ij}$ with $i, j \in \{1,2,3,4\}$ is the edge connected to the center vertex x. Without affecting its main characteristics, suppose $z_{ij} = z_{12}$, the set $W = \{z_{11}, z_{21}, z_{22}, z_{31}, z_{32}, z_{41}, z_{42}\}$ with 7 vertices and only one vertex that is not a member of W can share a vertex with that subdivision vertex e. Consider Figure 8 $e = z_{12}x$. Next, it will be shown that each vertex outside the set W has a unique representation, namely $\{z_{12}, z_{13}, x, z_{23}, z_{33}, z_{43}, e\}$. The seven vertices sequentially have unique representations of each other namely (2,2,2,2,2,2,2),(1,2,2,2,2,2,2,2),(1,1,1,1,1,1,1),(2,1,1,2,2,2,2),(2,2,2,1,1,2,2),(2,2,2,2,2,1,1),(1,3,3,3,3,3,3) This shows that $dim(S(G,e)) \leq 7$.

Both cases show that for every $e \in E(G)$ $dim(S(G,e)) \leq 7$ holds.

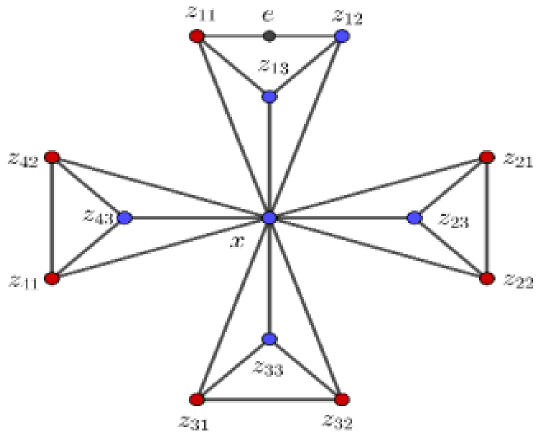

Figure 9. Pinwheel Graph $K_1 + 4K_3$

Next, it will be shown that $dim(S(G, e)) \geq 7$.Suppose that W is the distinguishing set of G. Based on *Lemma 1*, there are at least two vertices in D_2, D_3, D_4 that belong to the distinguishing set W. Next, it will be proved that there is at least one vertex in D_1 that also belongs to the distinguishing set W. Suppose there is no vertex in D_1 that belongs to the distinguishing set W, this means that all vertices in D_1have the same distance to all other vertices, so all vertices in D_1 have identical distances. This clearly creates a contradiction. Therefore, there is at least 1 vertex in D_1that belongs to the set W. Thus there are at least 7 vertices, namely at least two vertices from each of D_2, D_3, D_4 and at least 1 vertex from D_1 that belongs to the distinguishing set W. Consequently, the dimension of the distinguishing set $S(G, e)$ satisfies $dim(S(G, e)) \geq 7$

Consequently, based on inequalities (1) and (2), we obtain that $dim(S(G, e)) = 7$ for any subdivision edge in $G = K_1 + 4K_3$

CONCLUSION

Based on the research results, the metric dimension of the pinwheel subdivision graph $K_1 + mK_3$ with $2 \leq m \leq 4$ is 1 less than the metric dimension before subdivision, namely $dim\ (S(K_1 + mK_3, e) = dim(K_1 + mK_3) - 1$ For $2 \leq m \leq 4$. We encourage future researchers to find metric dimension on other graphs such as metric dimension on bridge subdivision graph with $m \geq 2$.

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Conflicts of Interest
The authors declare no conflict of interest.

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